

Predicting Arrival Delays: An Application of Spatial Analysis

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DOI: 10.2514/1.C031050

Analysts have many tools available to forecast delays. However, spatial analysis does not readily come to mind when predicting delays. Based on the case study of Newark Liberty International Airport, this study illustrates the potential applications of prevalent geostatistical techniques to delay forecast. There is a high degree of dependence among delays in a space or neighborhood defined by hour of operation and by day. Local spatial autocorrelation statistics help determine how delays are autocorrelated to other ones in a space. Among the other spatial analytical techniques, kriging enables the interpolation of delay estimates at unobserved spaces based on the values at observed spaces. Error estimates can be mapped to define spatial patterns (spaces where delays are likely to be more intense). Finally, spatial error regression provides a method for analysts to test the reliability of their findings when spatial dependency in errors is not taken into account. The study shows that delays in taxi operations at Newark Liberty International Airport, the volume of arrivals at Newark Liberty International Airport, as well as arrival demand at Newark Liberty International Airport and John F. Kennedy International Airport have a significant impact on the total minutes of arrival delays at Newark Liberty International Airport. This suggests that delay-reduction initiatives need to focus on demand management at Newark Liberty International Airport and LaGuardia Airport as well as on procedures to reduce taxi delays.

I. Introduction

THE airspace surrounding the New York City metropolitan area is among the busiest and most complex in the world. In 2008, the three largest New York City airports accounted for 1,273,145 takeoffs and landings according to the Federal Aviation Administration's Operations Network database (OPSNET). These airports have also stood among the most delayed facilities in the United States: According to Airline Service Quality Performance (ASQP) reports, 64.73% of the flights reported by the major domestic airlines in 2008 arrived on time at Newark Liberty International Airport (EWR), 70.85% at John F. Kennedy International Airport (JFK), and 66.85% at LaGuardia Airport (LGA). During the same time period, the national average was 77.76%. ASQP provides data on on-time performance, tarmac delays, and the causes of delays for a sample of 19 carriers at the time of this writing. These on-time statistics are compared with published airline schedules. For the Bureau of Transportation Statistics (BTS), a flight is delayed if it arrives 15 min or more past its scheduled gate-in time. In Aviation Systems Performance Metrics (ASPM) reports, delays can be measured as 1 min or more compared with the published schedule or flight plan (delay for all arrivals) or as 15 min or more (delay for delayed arrivals) based on the same criteria.

This study serves two purposes. First, it provides an illustration of how statistical techniques used in geostatistics and spatial econometrics can be applied to the forecast of delays as an alternative to more prevalent methods such as regression analysis. Second, it is meant to provide an introduction to spatial analysis for government, airline and airport analysts who need to evaluate the reliability of their models and findings.

Spatial analysis has been used in the airline sector to evaluate origin and destination flows (Derudder et al. [1] and Lee et al. [2]), market attributes and concentration mapping (Daraban and Fournier

[3], Reynolds-Feighan [4], and Spiller [5]). However, spatial analysis has not been used as an alternative methodology to forecast delays.

This study assumes that delay is a spatially distributed variable based on time of operations and day of the week (the space or neighborhood of a delay event). It focuses on spatial dependency in total minutes of arrival delays at EWR (i.e., the centroid). After describing the data and their sources, the discussion will proceed with the definition of the model variables. The analysis of the data structure through semivariograms and their dependency through the identification of spatial autocorrelation will lead to the interpolation of delays between sample points using kriging as an optimal interpolator. Finally, the exposition of spatial analytical techniques will end with a comparison between the ordinary least-squares regression and the spatial error models and identify differences in the model outcomes.

II. Data Analysis

A. Data Sample

The sample includes 558 instances of hours with arrival delays. The July 2009 data were organized by hour and by day. The choice of July was motivated by the high impact of convective weather on airport on-time performance and high volumes of traffic.

B. Data Sources

The operations, delay, and capacity metrics are stored and compiled in the ASPM data warehouse based on the following sources:

- 1) Out-off-on-in data are provided by Aeronautical Radio, Inc., (ARINC).
- 2) Runways' configuration and airport rate information are collected by the Air Traffic Control System Command Center (ATCSCC).
- 3) Air carriers' flight schedules are published by Innovata.
- 4) Flights records are assembled from the Enhanced Traffic Management System (ETMS).
- 5) On-time data compiled by the BTS in their monthly ASQP survey.

C. Definitions of Variables

Below are the definitions of all the model variables, including the sources of data:

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1) *Total minutes of arrival delays* represent the total delays of 1 min or more compared with the airline's gate-in schedule (sources: ASPM, ARINC, and Innovata).

2) *Weather* stands for the approach conditions at a particular hour based on ceiling and visibility minima (instrument-approach conditions in case of low visibility and ceiling *v.* visual-approach conditions in case of clear meteorological conditions). The periods in instrument-approach conditions are the hours when an airport is not capable of vectoring for visual approaches based on minima for ceiling and visibility. The minimum values are 3000 ft ceiling and 4 mile visibility at EWR, 2000 ft and 4 miles at JFK, and 3200 ft and 4 miles at LGA. The facilities provide the information to the ATCSCC.

3) *Runway configurations* are the set of arrival and departure runways in use at a particular hour (source: ATCSCC).

4) *Wind angle* is the direction of the wind in degrees (source: National Oceanic and Atmospheric Administration).

5) *Total minutes of taxi-in delays* at EWR are the difference between the actual minutes of taxiing from wheels-on to gate-in and an impeded taxi-in times computed in ASPM based on ASQP carriers and season (source: ASPM, ARINC, ASQP).

6) *Total minutes of taxi-out delays* at EWR represent the additional minutes of taxiing from gate-out to wheels-off compared with unimpeded taxi-out times (source: ASPM, ARINC, ASQP).

7) *Arrival demand* is computed on an individual flight basis within 15 min increments. Arrival demand starts at the time when aircraft wheels are off and includes the estimated time en route. It ends when wheels are on (sources: ASPM and ARINC).

8) *Arrivals and departures* are the volume of traffic handled by the EWR facility (source: ETMS).

9) *Total minutes of airport departure delays* include the gate departure delays in minutes (actual gate-out time compared with scheduled gate-out time) plus taxi-out delays as explained above (sources: ETMS, ARINC, and Innovata).

10) *Total minutes of airborne delays*. Airborne delay is the actual airborne time minus a carrier's submitted estimated time en route (sources: ASPM and ETMS).

11) *Total minutes of taxi-in* account for the total minutes of operations from wheels-on to gate-in (sources: ARINC and ETMS).

D. Data Processing

The data were input into SAS®. The MIXED procedure enabled the selection of the covariate structure based on the lowest value of the fit statistics (residual log likelihood, Akaike's and Schwarz's criteria). The VARIOGRAM procedure made it possible to generate the theoretical and empirical semivariograms. Kriging analysis was performed with the KRIEGE2D procedure. The REG procedure provided regression analysis.

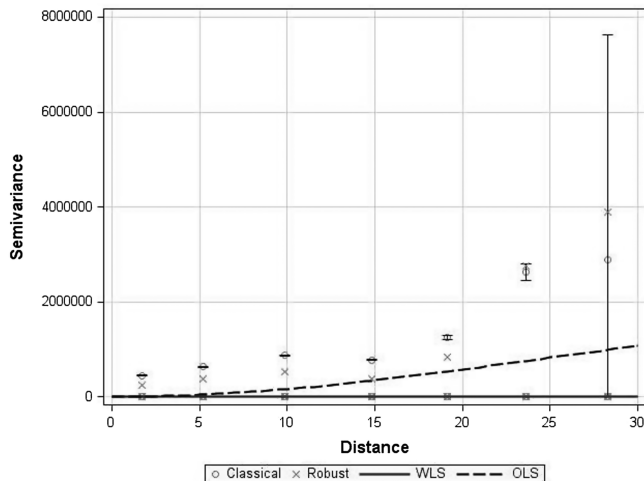


Fig. 1 Empirical and fitted theoretical semivariograms.

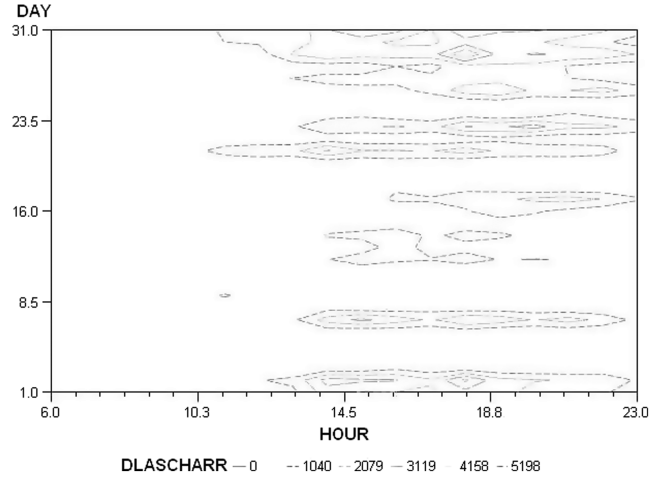


Fig. 2 Contour plot of arrival delay minutes (by hour and by day).

E. Measurement of Spatial Dependency

The variogram is a tool for quantifying spatial correlation (see Appendix A for further details). It displays the variances within groups of observations plotted as a function of distance between observations. The variogram is also used to examine the degree of spatial continuity in data at various lags or distances of separation, assuming the data are stationary.

Semivariance represents half the average squared difference between pairs of arrival delays at a given distance apart. The semivariogram measures variance among sites as a function of distance. Figure 1 provides a comparison between the empirical and the theoretical semivariograms (see Cressie [6]). In geostatistics, semivariograms are more readily used than covariograms, which express dependency between random variables in terms of covariances. The comparison of the fit statistics determined that the Gaussian covariance structure was preferable to others (spherical or exponential, for instance).

The semivariograms suggest that points separated by short distances are correlated (Fig. 1). The correlation decreases as the distance between points increases (up to a distance of 15 in the present case). Because the empirical variogram provides estimates for a definite set or classes of lags, it is necessary to fit a parametric semivariogram in order to derive large-scale trends and provide spatial predictions. Other forms of fit are also indicated in Fig. 1: weighted least-squares and ordinary least-squares (OLS). Figure 2 provides the duration, intensity, and days when significant arrival delays happened.

III. Spatial Analysis

Spatial analysis refers to a set of techniques applied to autocorrelation, interpolation, regression and interaction models. After calculating the empirical semivariogram and modeling the theoretical variogram, the next step consists in identifying spatial autocorrelation and then using ordinary kriging to identify the best linear unbiased predictors and errors. Finally, we will focus on one type of spatial regression based on the error as opposed to a lagged dependent variable.

A. Spatial Autocorrelation

The pattern of spatial distribution can be measured with two statistics:

1) Moran's I is computed as

$$I(d) = \frac{\sum_i^n \sum_j^n w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{\left(S^2 \sum_i^n \sum_j^n w_{ij} \right)} \quad (1)$$

where

Table 1 Autocorrelation statistics for selected variables (source: ASPM)

Assumption	Coefficient	Observed	Expected	Std dev	Z	Pr > Z
<i>Total minutes of arrival delays</i>						
Normality	Moran's <i>I</i>	0.0825	-0.0018	0.0018	46.9	<0.0001
Normality	Geary's <i>c</i>	0.8649	1	0.0074	-18.2	<0.0001
<i>EWR arrivals</i>						
Normality	Moran's <i>I</i>	0.069	-0.0018	0.0018	39.4	<0.0001
Normality	Geary's <i>c</i>	0.909	1	0.0074	-12.3	<0.0001
<i>Actual taxi-in times</i>						
Normality	Moran's <i>I</i>	0.0875	-0.0018	0.0018	49.6	<0.0001
Normality	Geary's <i>c</i>	0.8873	1	0.0074	-15.2	<0.0001
<i>EWR departure demand</i>						
Normality	Moran's <i>I</i>	0.0277	-0.0018	0.0018	16.41	<0.0001
Normality	Geary's <i>c</i>	0.9624	1	0.0074	-5.06	<0.0001
<i>EWR arrival demand</i>						
Normality	Moran's <i>I</i>	0.0855	-0.0018	0.0018	48.5	<0.0001
Normality	Geary's <i>c</i>	0.8475	1	0.0074	-20.5	<0.0001
<i>JFK arrival demand</i>						
Normality	Moran's <i>I</i>	0.0793	-0.0018	0.0018	45.1	<0.0001
Normality	Geary's <i>c</i>	0.8798	1	0.0074	-16.2	<0.0001
<i>LGA arrival demand</i>						
Normality	Moran's <i>I</i>	0.0503	-0.0018	0.0018	29	<0.0001
Normality	Geary's <i>c</i>	0.8958	1	0.0074	-14	<0.0001

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (2)$$

2) The formula for Geary's *c* is

$$c(d) = (n-1) / \left(2 \sum_i \sum_j w_{ij} \right) \left\{ \sum_i \sum_j w_{ij} (x_i - x_j)^2 / \sum_i (x_i - \bar{x})^2 \right\} \quad (3)$$

Moran's *I* is a parametric test that measures how related the values of a variable are based on the location where they are measured. Correlation is weighted by the inverse distance and it is computed as $1/(1 + \text{distance between points A and B})$. The test indicates whether spatial correlation is positive or negative and it provides a *p* value for the level of autocorrelation. A positive value of Moran's *I* suggests that observations tend to be similar, contrary to a negative value. The observations are arranged randomly over space when Moran's *I* is close to zero. Moran's *I* tests against the null hypothesis that there is no spatial autocorrelation.

Based on the *p* values of the reported Moran's *I* featured in Table 1, we can reject the null hypothesis that there is zero spatial autocorrelation in the values of the specified variables, since all the *p* values are less than $\alpha = 0.10$.

A Geary's *c* value that is greater than or equal to 1 means that observations are likely to be dissimilar, as opposed to a *c* value less than 1. Geary's *c* varies from 0 (maximum positive autocorrelation)

to a positive value for high negative autocorrelation. A score of 1 means there is no autocorrelation. If the value is less than 1, then spatial autocorrelation is positive as it is the case for all the variables in Table 1. The distance matrix used in this study is 558×558 observations.

B. Spatial Interpolation: Ordinary Kriging

After identifying the spatial correlation structure of a variable, the data from the measured locations can be used to estimate a variable at locations where it had not been measured. This interpolation from measured locations or spaces to unmeasured locations is referred to as kriging. The value of a point x^* is estimated as a linear combination such that $f(x^*) = \sum_{i=1}^n \lambda_i(x^*) f(x_i)$ with the weight λ_i as solutions of a system of linear equations.

There are three steps in kriging analysis. The first one consists in computing the empirical variogram of the sampled data. Second, a theoretical variogram that fits the sample variogram for spatial extrapolation is selected among different mathematical forms and parameters (exponential, Gaussian, spherical, and power). Third, the variogram is used to solve the kriging system at a specified set of spatial points.

In ordinary kriging, the variable values from the measured locations become covariates for the values at unmeasured locations. In the present ordinary kriging model, the coordinates used to determine the location of points in the grid are days and hours.

Local kriging is performed by using only data points within a specified radius of each grid point. There are 49 prediction grid points and the neighborhood is based on a radius of 30 days to fully reflect

Table 2 Kriging estimates and standard errors of arrival delay minutes

Observations	GXC ^a (day)	GYX ^b (hour)	Number of points ^c	Estimates	Standard error
1	0	0	452	-22.6430	1.7837
2	0	5	527	129.6150	1.6430
3	0	10	555	92.3830	1.6197
4	0	15	558	24.7340	1.6179
5	0	20	558	35.7970	1.6176
6	0	25	543	105.1210	1.6283
7	0	30	488	75.0180	1.7001
8	5	0	492	68.6300	0.9255
9	5	5	552	38.5180	0.6881
10	5	10	558	102.6250	0.6874

^aGXC is the *x* coordinate of the grid point at which the kriging estimate is made.

^bGYC is the *y* coordinate of the grid point at which the kriging estimate is made.

^cThe number of points are those used for each estimate. The geographic *X* coordinate refers to day and the geographic *Y* coordinate is for hour.

the spatial correlations in the data. The type of covariance model is spherical with a sill of 6.5 and a range of 30. The nugget effect is 0. The nugget effect is the resulting discontinuity in the semivariograms at the origin. The relative nugget effect is the percentage of the overall variance attributable to measurement error.

For each predicted location, the variance-covariance structure identified in the variogram is used to fit a linear function to these covariates. As a result, it is possible to derive an unbiased linear prediction (estimates) and a standard error of the value at the location detail as shown in Table 2. Predictions are derived as weighted averages of known values. The points closer to the one to interpolate have a greater weight. Clusters of points become a single point. Finally, the error of each prediction is based on the sample point locations.

C. Spatial Error Model

The MIXED procedure in SAS was used to fit a regression model in which the results and expected errors are spatially autocorrelated. The spatial error model for any i observation (location) can be written as

$$y_i = \mathbf{x}_i^* \beta + \lambda^* \mathbf{w}_i^* \xi_i + \varepsilon_i \quad (4)$$

where \mathbf{w}_i is a vector of \mathbf{W} that denotes the proximity of units with each other, ξ_i the spatial component of the error term and ε_i a spatially uncorrelated error term. The OLS implies that the error term is homoscedastic and has no autocorrelation: $\text{var}(\varepsilon_i) = \sigma^2$, $\text{cov}(\varepsilon_i, \varepsilon_j) = 0$ and $E[\varepsilon \varepsilon'] = \sigma^2 \mathbf{I}$. A spatial error model can be considered as a regression with a nonspherical error term.

The three models were selected on the basis of how they, respectively, achieved the lowest value of the Akaike information criterion, Akaike information corrected criterion, and Bayesian information criterion. The outcomes of three models were compared in order to assess the significance of the variables in predicting arrival delay minutes at EWR, as well as the impact of fixed- and random-effect variables.

In spatial analysis, there are two possible types of effects. The fixed effect is characterized by spatial dependence/autocorrelation. This implies that there is a functional relationship between what

happens at one point in space and what happens elsewhere. The random effect refers to spatial heterogeneity. In this case, the parameters vary with location and are not homogeneous throughout the data set.

The first spatial error model is characterized by spatial Gaussian covariance and an intercept subject effect (Fig. 3). The outcomes indicate that the arrivals and arrival demand at EWR, as well as the arrival demand at JFK, all have a significant effect on the arrival delay minutes at EWR (based on schedule). Despite the proximity of LGA and EWR in physical distance, the arrival demand at LGA does not have a significant fixed effect on the arrival delay minutes at EWR.

In the second spatial error model (Fig. 4), the dependent variable is the total minutes of arrival delays minutes at EWR, while the explanatory variables can be classified in three main groups:

1) Class-level information includes runway configurations at EWR, meteorological conditions, runway configurations at JFK, runway configurations at LGA, wind speed, and wind angle.

2) Random effects include minutes of taxi-in delays, minutes of taxi-out delays, and arrival demand at EWR.

3) Fixed effects include arrivals at EWR, departures from EWR, minutes of airport departure delays, minutes of airborne delays, minutes of taxi-in time, arrival demand at JFK, arrival demand at LGA, wind speed, wind angle, and meteorological conditions.

The second spatial error model is a mixed model, since it also includes random effects that EWR may find it difficult to anticipate and to control. The random effects can also be considered as triggering devices for delays and congestion as they affect airport capacity and throughput.

The second spatial error model shows lower values for the fit statistics (Akaike information criterion, Akaike information corrected criterion, Bayesian information criterion, and $-2 \log$ likelihood) than the first one, which is preferable in selecting a model selection. The second model also implies that the random effects of taxi-in/out delay and arrival demand at EWR are significant at a 90% confidence level. These variables are indicators of airport congestion, most likely to impact delays. However, the fixed effects of airborne

Covariance Parameter Estimates		
Covariance Parameter	Subject	Estimate
Spherical (Gaussian)	Intercept	0.8073
Residual		132491

Fit Statistics	
-2 Res Log Likelihood	8092.70
AIC ¹	8096.70
AICC ²	8096.70
BIC ³	8105.30

(1) Akaike Information Criterion
(2) Akaike Information Corrected Criterion
(3) Bayesian Information Criterion

Null Model Likelihood Ratio Test		
DF	Chi-Square	Pr > ChiSq
1	60	<.0001

Solution for Fixed Effects					
Effect	Estimate	Standard Error	DF	t Value	Pr > t
Intercept	278	63.9079	0	4	.
Arrival at EWR	-23	2.0115	553	-11	<.0001
Arrival Demand at EWR	16	0.7646	553	21	<.0001
Arrival Demand at JFK	3	0.7818	553	4	<.0001
Arrival Demand at LGA	1	0.7607	553	1	0.1947

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Arrivals to EWR	1	553	126.24	<.0001
Arrival Demand at EWR	1	553	451.89	<.0001
Arrival Demand at JFK	1	553	16.99	<.0001
Arrival Demand at LGA	1	553	1.69	0.1947

Fig. 3 Exhibit 1: spatial error model 1.

Covariance Parameter Estimates		Fit Statistics			
Covariance Parameter	Estimate	-2 Log Likelihood	7893.5		
Taxi-In Delay	697.48	AIC	8,025.5		
Taxi-Out Delay	0.10	AICC	8,043.5		
Arrival Demand	196.41	BIC	7,893.5		
Spherical (Gaussian)	1.00				
Residual	79,128.00				

Solution for Random Effects					
Effect	Estimate	Std Error Predicted	Degree Freedom	t Value	Pr > t
Taxi-In Delay	-24.94	13.105	493.00	-1.90	0.06
Taxi-Out Delay	-0.32	0.051	493.00	-6.29	<.0001
Arrival Demand	14.01	0.66	493.00	21.23	<.0001

Effect	Alpha	Lower	Upper
Taxi-In Delay	0.90	-26.58	-23.2893
Taxi-Out Delay	0.90	-0.33	-0.3149
Arrival Demand	0.90	13.93	14.0956

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Departures from EWR	1	493	7.70	0.0057
Arrivals to EWR	1	493	124.40	<.0001
Airport Departure Delays	1	493	59.33	<.0001
Airborne Delays	1	493	1.76	0.1852
Taxi-In Times	1	493	52.90	<.0001
Arrival Demand at JFK	1	493	2.12	0.1458
Arrival Demand at LGA	1	493	0.02	0.8874
Windspeed	16	493	4.54	<.0001
Windangle	36	493	1.60	0.0169
Meteorological Conditions ^a	1	493	8.64	0.0034

(a) Meteorological conditions refer to visual or instrument approach conditions based on ceiling and visibility minima at the airport.

Fig. 4 Exhibit 2: spatial error model 2.

Number of Observations Read	558				
Number of Observations Used	558				
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	306,859,914	43,837,131	198.74	<.0001
Error	550	121,318,912	220,580		
Corrected Total	557	428,178,826			
Root MSE	469.659	R-Square	0.7167		
Dependent Mean	718.416	Adj R-Sq	0.7131		
Coefficient of Variance	65.3743				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	122.09	86.93	1.40	0.1608
Departures from EWR	1	-14.51	2.14	-6.78	<.0001
Arrivals to EWR	1	-8.82	3.17	-2.78	0.0056
Airport Departure Delays	1	0.25	0.03	9.49	<.0001
Airborne Delays	1	0.45	0.17	2.71	0.0069
Taxi-In Times	1	0.68	0.34	2.01	0.0450
Arrival Demand at JFK	1	7.69	0.87	8.80	<.0001
Arrival Demand at LGA	1	9.04	0.74	12.20	<.0001

Fig. 5 Exhibit 3: ordinary least-squares model.

delay, arrival demand at *both* JFK and LGA are not significant at 90% confidence level. This may be counterintuitive in an airspace environment where the Terminal Radar Approach Control (TRACON) affects each airport's capacity utilization and on-time performance by managing demand for arrivals and departures. Among the fixed effects, wind angle plays a significant role on airport efficiency and capacity utilization, because it determines the optimal runway configuration. Sudden changes in wind angle may affect the efficiency of ground operations that translate into taxi-in and taxi-out delays. Since the three New York airports are relatively close to 1 another, the choice of a configuration at one airport is likely to affect how arrival demand is handled at the two others.

Figure 5 provides the estimates from an OLS regression model computed with the REG procedure in SAS.

In Fig. 5, only the intercept is not significant at 90% confidence level. Contrary to the spatial error models 1 and 2, the arrival demand at JFK and LGA are all significant at a 90% confidence level. As a result, a policy maker may focus on a solution that accommodates the three airports as a whole instead of focusing on the management of arrival demand where it is significant (as the spatial error models 1 and 2 suggested).

IV. Conclusions

Spatial analysis is a relatively new methodology in the airline industry, while it is a mainstay in geostatistics. Spatial analysis assumes that delays represent a location in a space determined by coordinates such as hour of operation and day in the present case.

This paper was designed to illustrate how spatial analytical methods can be applied to the study of delay and its forecast. Kriging can be instrumental in identifying clusters of delays and their mapping, to interpolate estimates and to determine errors. The mapping can be based on other coordinates such as runway configurations or sections of the airport.

Finally, the results show that analysts have to be careful when evaluating the factors likely to impact delays. Variables that are significant in an OLS model may not be so in spatial error models. In this study, the outcomes of the spatial error models point out to the significance of internal operational factors such as taxi delays and demand management in reducing delays at EWR. The finding that arrivals and/or arrival demand at nearby alternate airports may influence delays supports the ongoing efforts to redesign the New York area airspace. This involves combining high and low altitude airspace to create more efficient arrival and departure routes.

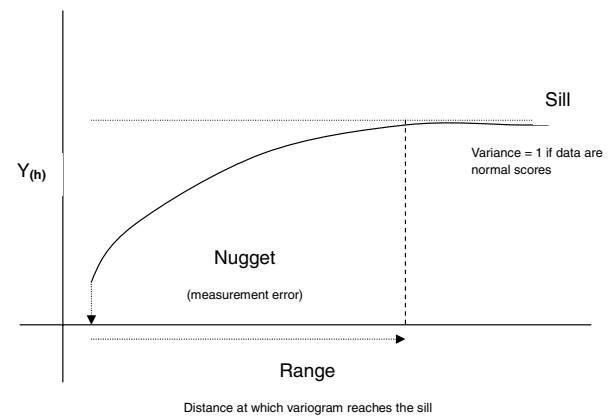


Fig. A1 Key components of the empirical variogram.

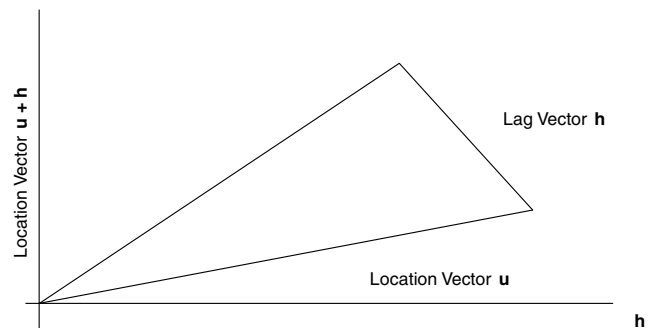


Fig. A2 Location of the key vectors.

Appendix A: Variogram/Semivariogram

Figure A1 shows the key components of the empirical variogram/semivariograms and Fig. A2 indicates the location of the different vectors. In Fig. A1, the range is the point on the x axis, where the curve reaches a plateau. The sill is the height of the curve at the plateau. In Fig. A2, the lag is the difference between two sites in a pair. The number of pairs available for computing a semivariogram depends on the lag distance.

When spatial correlation is stronger in one direction than the other, the spatial correlation pattern is *anisotropic*. Anisotropy can be geometric or zonal. Geometric anisotropy can usually be corrected by linear transformation. When the spatial correlation depends on the distance and not the direction of separation, the spatial correlation pattern is characterized as *isotropic*. The lag tolerance is chosen to be *half of the smallest lag spacing*. As a result, no pair of points will be used for more than one lag distance.

The theoretical variogram in this study is Gaussian and can be characterized as

$$\gamma_z(h) = c_0[1 - \exp(-h^2/a_0^2)] \quad (A1)$$

Acknowledgment

This research does not reflect the opinion of the Federal Aviation Administration or the Office of Aviation Policy and Plans.

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